Notes for the meeting on 14-02-2013 [redacted]

[redacted]

1. $$S = k_B \ln \frac{N!}{N_\uparrow !\, N_\downarrow !} \simeq k_B \left[N \ln \frac{N}{e} - N_\uparrow \ln \frac{N_\uparrow}{e} - N_\downarrow \ln \frac{N_\downarrow}{e} \right]$$

$$N_\uparrow + N_\downarrow = N$$

$$E = mg Z_2 = mga (N_\uparrow - N_\downarrow) = mga (2N_\uparrow - N)$$

$$dS = k_B \left[- \ln N_\uparrow \, dN_\uparrow - \ln N_\downarrow \, \underset{-dN_\uparrow}{\underset{\parallel}{dN_\downarrow}} \right] = k_B \ln \frac{N_\downarrow}{N_\uparrow} \, dN_\uparrow$$

chain + load

$$dE = 2mga \, dN_\uparrow$$

$$\frac{1}{T} = \frac{dS}{dE} = \frac{k_B}{2mga} \ln \frac{N_\downarrow}{N_\uparrow} \; ; \qquad \frac{N_\downarrow}{N_\uparrow} = e^{\frac{2mga}{k_B T}}$$

$$N_\downarrow = N \frac{e^{\beta mga}}{e^{\beta mga} + e^{-\beta mga}} \; , \qquad N_\uparrow = N \frac{e^{-\beta mga}}{e^{\beta mga} + e^{-\beta mga}}$$

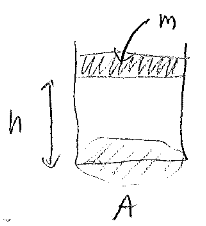
$$Z_2 = a (N_\uparrow - N_\downarrow) = - aN \tanh(\beta mga)$$

If $\beta mga \ll 1$, i.e. $mga \ll k_B T$ $\qquad Z_2 \approx - \frac{N mga^2}{k_B T}$

Another way:

$$dE_{chain} = T dS + f \, dL = T dS - mg \, dZ_2 = 0 \qquad T dS = mg \, dZ_2$$

$$f = mg \, , \; L = -Z_2$$



$$W = E + mgh = E + \frac{mg}{A} V = E + PV$$

$$dW = dE + mg \, dh = dE + \frac{mg}{A} dV = dE + p dV$$

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distribution of grades

UV	6	6.5	7	7.5	8	8.5	9	9.5	10	V	total
	𝍸 |||	||||	𝍸	|||	𝍸 ||							𝍸 ||		
21	8	4	5	3	7	1	1	1	7	37	58

2.

a) $$Z = \int \frac{d^3x\, d^3p}{h^3} e^{-\beta c p} = \frac{4\pi V}{8\pi^3\hbar^3} \int_0^\infty dp\, p^2 e^{-\beta c p} =$$

$$= \frac{4\pi V}{8\pi^3\hbar^3} \left(\frac{k_B T}{c}\right)^3 \int_0^\infty dx\, x^2 e^{-x} = \frac{V}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3$$

b) $$\mathcal{Z} = \frac{Z^N}{N!}, \quad F = -k_B T \ln \mathcal{Z} \approx -N k_B T \ln \frac{eZ}{N}$$

$$S = -\frac{\partial F}{\partial T} = N k_B \left[\ln \frac{eZ}{N} + 3 \right] = N k_B \left[\ln \frac{Z}{N} + 4 \right]$$

c) $$E = -\frac{\partial}{\partial \beta} \ln \mathcal{Z} = \frac{3N}{\beta} = 3 N k_B T$$

$$P = -\left(\frac{\partial F}{\partial V}\right)_{T,N} = \frac{N k_B T}{V} \quad \Rightarrow \quad PV = \frac{1}{3} E$$

another way $$F = E - TS \quad \Rightarrow \quad E = F + TS = -N k_B T \ln \frac{eZ}{N} +$$

$$+ N k_B T \left[\ln \frac{eZ}{N} + 3 \right] = 3 N k_B T$$

d) $$C_v = \frac{1}{N} T \left(\frac{\partial S}{\partial T}\right)_{V,N} = \frac{1}{N}\left(\frac{\partial E}{\partial T}\right)_{V,N} = 3 k_B$$

$$C_p = \frac{1}{N} T \left(\frac{\partial S}{\partial T}\right)_{P,N} = \frac{1}{N}\left(\frac{\partial W}{\partial T}\right)_{P,N} = \frac{4}{3} C_v = 4 k_B$$

$$W = E + PV = \frac{4}{3} E$$

$$\gamma = \frac{C_p}{C_v} = \frac{4}{3}$$

$$S = N k_B \left[\ln \frac{Z}{N} + 4 \right] = N k_B \left[\ln \left(\frac{V}{\pi^2 N} \left(\frac{k_B T}{\hbar c}\right)^3 \right) + 4 \right]$$

$$= N k_B \left[\ln \frac{\cancel{N} k_B T}{\pi^2 P \cancel{N}} \left(\frac{k_B T}{\hbar c}\right)^3 + 4 \right]$$

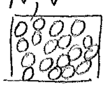
$$C_p = \frac{1}{N} T \left(\frac{\partial S}{\partial T}\right)_{P,N} = \frac{1}{N} T\, N k_B \frac{4}{T} = 4 k_B$$

3.  $E = \frac{3}{2} N k_B T \qquad C_v = \frac{1}{N}\left(\frac{\partial E}{\partial T}\right)_{V,N} = \frac{3}{2} k_B$

 $C_v = \frac{3}{2} k_B + 2 \cdot \frac{1}{2} k_B = \frac{5}{2} k_B$

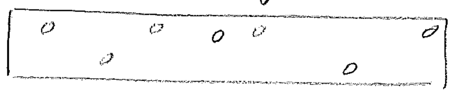
 $C_v = \frac{3}{2} k_B + 3 \frac{1}{2} k_B = 3 k_B$

4. N, V



$T=0$

V'



$p < p_F$

a) $$\langle \varepsilon \rangle = \frac{E}{N} = \frac{2 \sum_{\vec{p}}' \varepsilon_{\vec{p}}}{2 \sum_{\vec{p}}' 1} = \frac{\int_0^{p_F} dp\, p^2 \frac{p^2}{2m}}{\int_0^{p_F} dp\, p^2} = \frac{\frac{1}{5} \frac{p_F^5}{2m}}{\frac{1}{3} p_F^3} = \frac{3}{5} \frac{p_F^2}{2m}$$

$$\langle \varepsilon \rangle = \frac{3}{5} \varepsilon_F \qquad \boxed{E = \frac{3}{5} \varepsilon_F N}$$

b) $$PV = \frac{2}{3} E = \frac{2}{3} \frac{3}{5} \varepsilon_F N = \frac{2}{5} \varepsilon_F N \qquad P = \frac{2}{5} \varepsilon_F \frac{N}{V} = \frac{2}{5} \varepsilon_F n$$

$$\left(\Omega = -PV = -\frac{2}{3} E \right)$$

$$P = -\left(\frac{\partial E}{\partial V} \right)_N$$

$$E \propto \varepsilon_F \propto p_F^2 \propto n^{+2/3} \propto V^{-2/3}$$

$$\left(\frac{\partial E}{\partial V} \right)_N = -\frac{2}{3} \frac{E}{V} \qquad P = \frac{2}{3} \frac{E}{V}$$

c) $$N' = N, \quad \underset{\frac{3}{2} P'V'}{\underset{\parallel}{E'}} = \underset{\frac{3}{2} PV}{\underset{\parallel}{E}} \Rightarrow P'V' = PV, \quad P' = P \frac{V}{V'}$$

$$E = \frac{3}{2} N k_B T' = \frac{3}{5} \varepsilon_F N \Rightarrow k_B T' = \frac{2}{5} \varepsilon_F = \frac{2}{5} k_B T_F, \quad T' = \frac{2}{5} T_F$$

$$P'V' = N k_B T' = \frac{2}{3} \cdot \frac{3}{2} N k_B T' = \frac{2}{3} \cdot \frac{3}{5} \varepsilon_F N = \frac{2}{5} \varepsilon_F N =$$
$$= \frac{2}{5} \varepsilon_F \frac{N}{V} V = PV$$

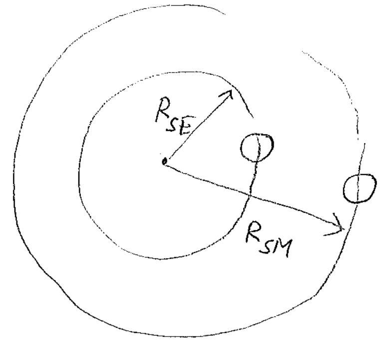
$$S = 0, \quad F' = -k_B T \ln Z = -N k_B T' \ln \frac{eZ}{N} = -N k_B T' \ln \frac{eV'}{N \lambda^3}$$
$$\lambda = h / \sqrt{2\pi m k_B T'}$$

$$S' = -\left(\frac{\partial F'}{\partial T'} \right)_{V', N} = N k_B \left[\ln \frac{eV'}{N \lambda^3} + \frac{3}{2} \right]$$

$$\frac{V'}{N} = a^3 \qquad S' = N k_B \left[3 \ln \frac{a}{\lambda} + \frac{5}{2} \right]$$

$a \gg \lambda$ (otherwise the gas is not classical) $\Rightarrow S' > 0$

5.



$$L \cdot \frac{\pi R_E^2}{4\pi R_{SE}^2} = \sigma T_E^4 \cdot 4\pi R_E^2$$

$$T_E^4 = \frac{L}{16\pi\sigma \, R_{SE}^2} \quad , \quad T_M^4 = \frac{L}{16\pi\sigma \, R_{SM}^2}$$

$$\left(\frac{T_M}{T_E}\right)^4 = \left(\frac{R_{SE}}{R_{SM}}\right)^2 \quad , \quad \frac{T_M}{T_E} = \left(\frac{R_{SE}}{R_{SM}}\right)^{1/2} = \left(\frac{2}{3}\right)^{1/2}$$